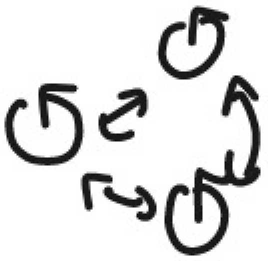
KURAMOTO MODEL

$\dot{\Theta} = \omega \qquad \Theta(t) = \Theta_0 + \omega t$



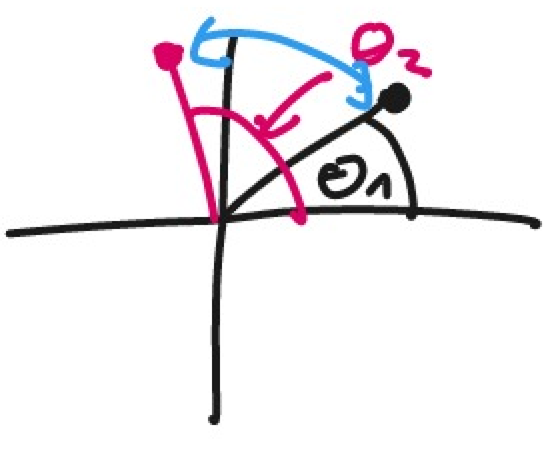
$\dot{\Theta}_n = \omega_n \qquad n = 1, 2, \dots, N$

$$\boxed{\dot{\Theta}_n = \omega_n + f_n(\Theta_1, \dots \Theta_n, \dots \Theta_N)}$$

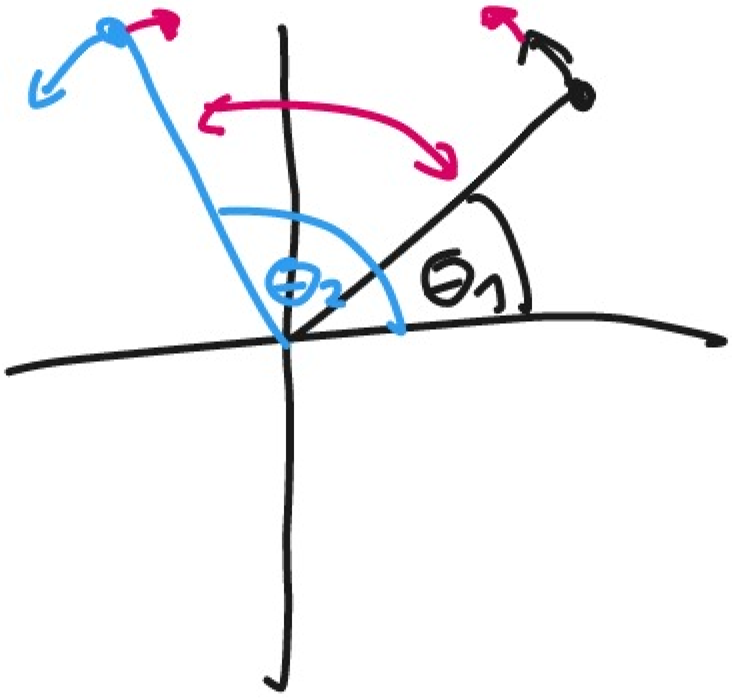
$$\dot{\Theta}_1 = \omega_1 + f_1(\Theta_1, \Theta_2)$$
$$\dot{\Theta}_2 = \omega_2 + f_2(\Theta_1, \Theta_2)$$

$f_1 = f_2 = f$

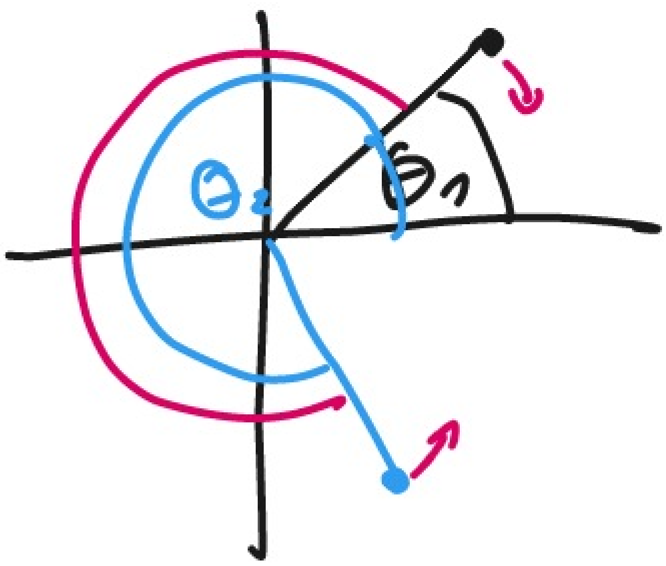
$$\dot{\Theta}_1 = \omega_1 + f(\Theta_1, \Theta_2)$$
$$\dot{\Theta}_2 = \omega_2 + f(\Theta_2, \Theta_1)$$



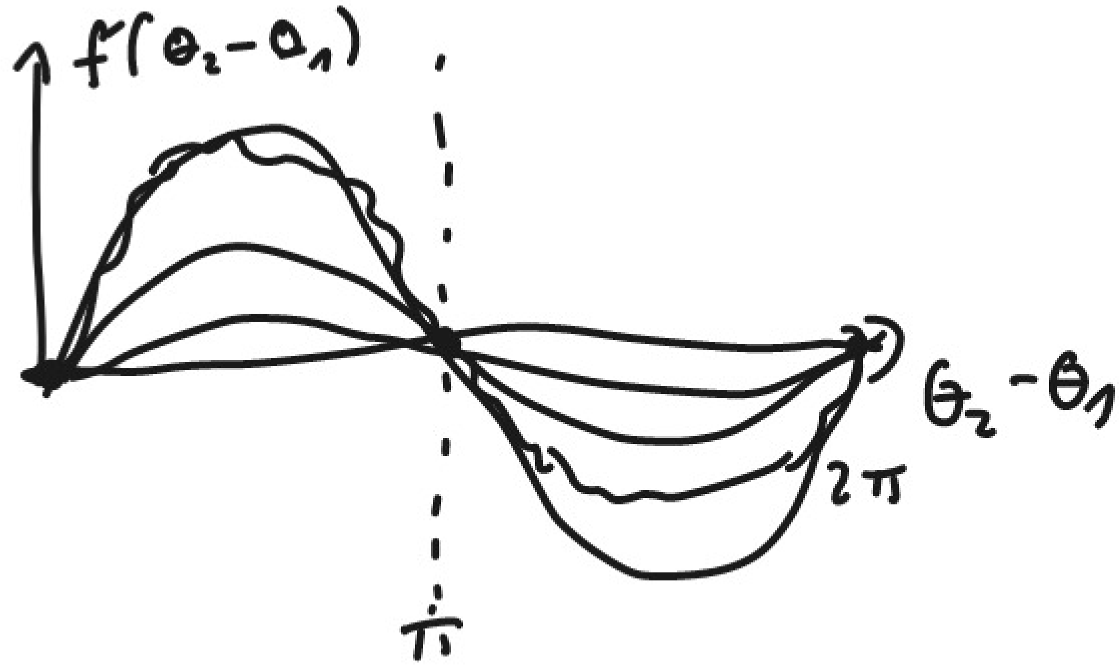
$$\boxed{\begin{aligned} \dot{\Theta}_1 &= \omega_1 + f(\Theta_2 - \Theta_1) \\ \dot{\Theta}_2 &= \omega_2 + f(\Theta_1 - \Theta_2) \end{aligned}}$$



$\theta_2 - \theta_1 < \pi \qquad f(\theta_2 - \theta_1) > 0$



$\theta_2 - \theta_1 > \pi \qquad f(\theta_2 - \theta_1) < 0$



$$\boxed{\begin{aligned} \dot{\theta}_1 &= \omega_1 + \frac{K}{2}\sin(\theta_2 - \theta_1) \\ \dot{\theta}_2 &= \omega_2 + \frac{K}{2}\sin(\theta_1 - \theta_2) \end{aligned}}$$

$$x = \theta_2 - \theta_1$$

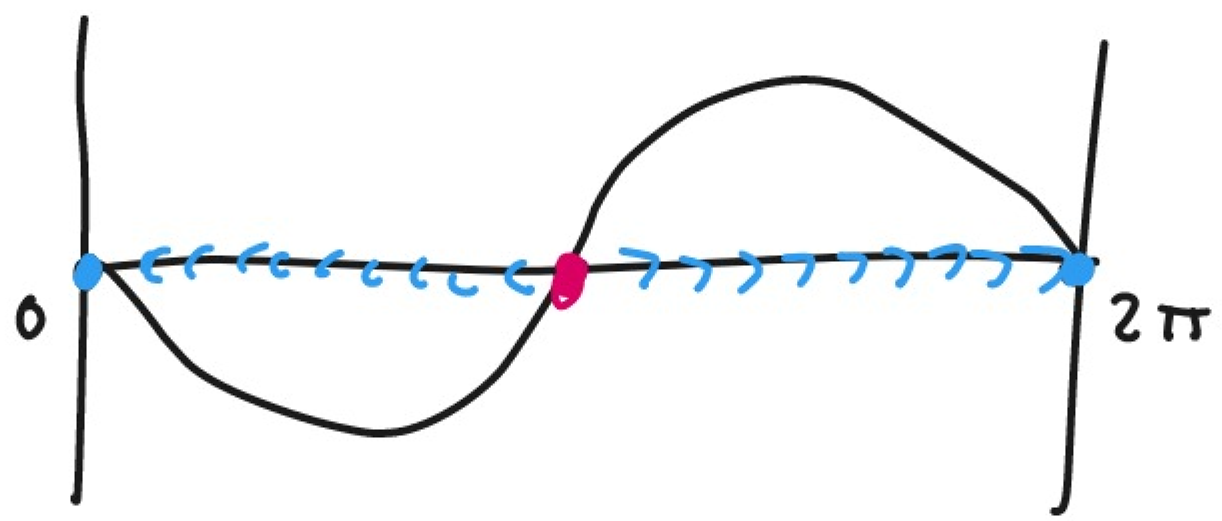
$$\dot{x} = \dot{\theta}_2 - \dot{\theta}_1$$

$$= \underbrace{\omega_2 - \omega_1}_{\delta\omega} + \frac{K}{2}\left[\underbrace{\sin(-x)} - \sin(x)\right]$$

$$\boxed{\dot{x} = \delta\omega - k\sin(x)}$$

$\delta\omega = 0 \quad (\omega_2 = \omega_1)$

$\dot{x} = -k\sin(x) = g(x) \qquad k > 0$

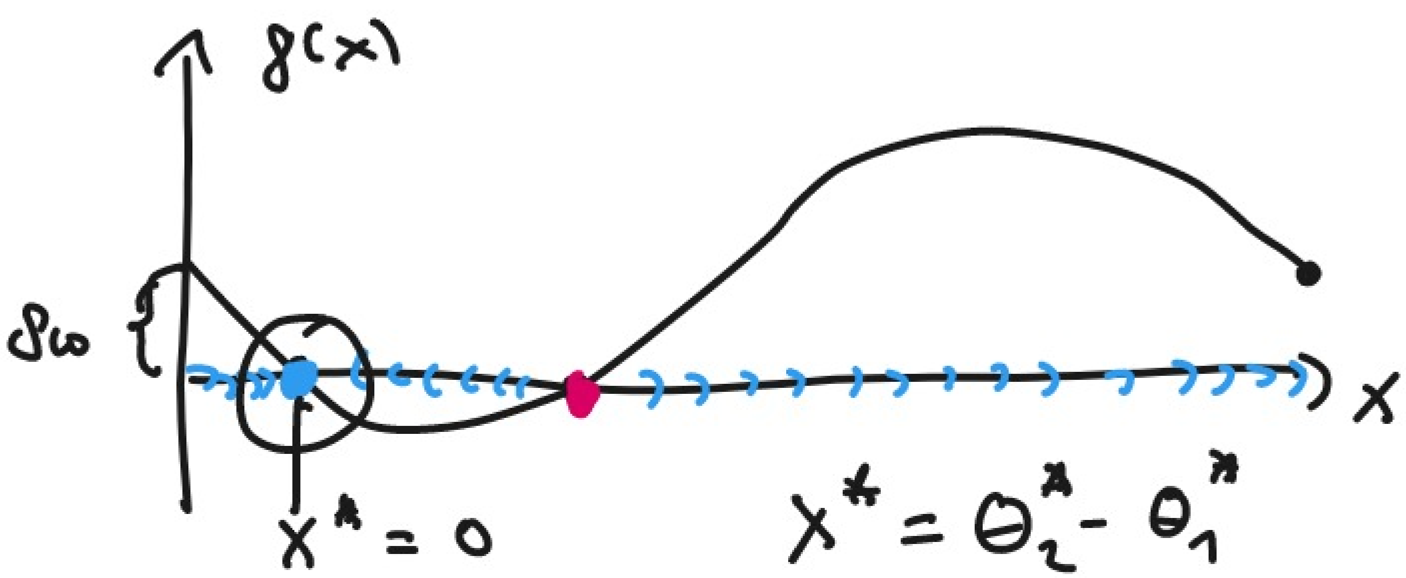


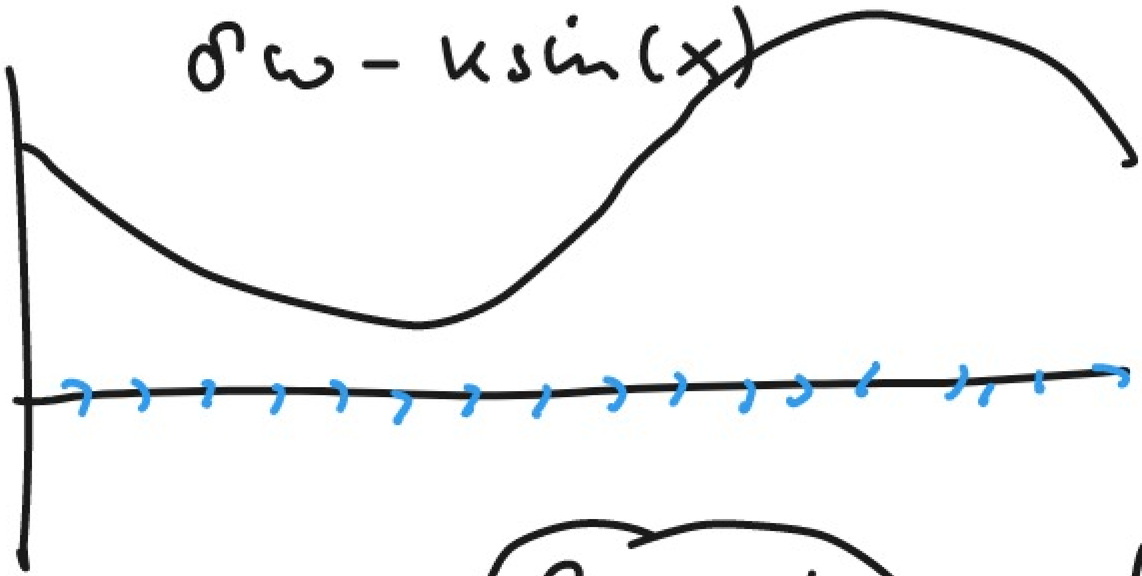
$x^* = 0 \quad (x = \theta_2 - \theta_1)$

$\delta\omega > 0$

$\dot{x} = \delta\omega - k\sin(x) = g(x)$

$x^* = \sin^{-1}(\delta\omega / k)$





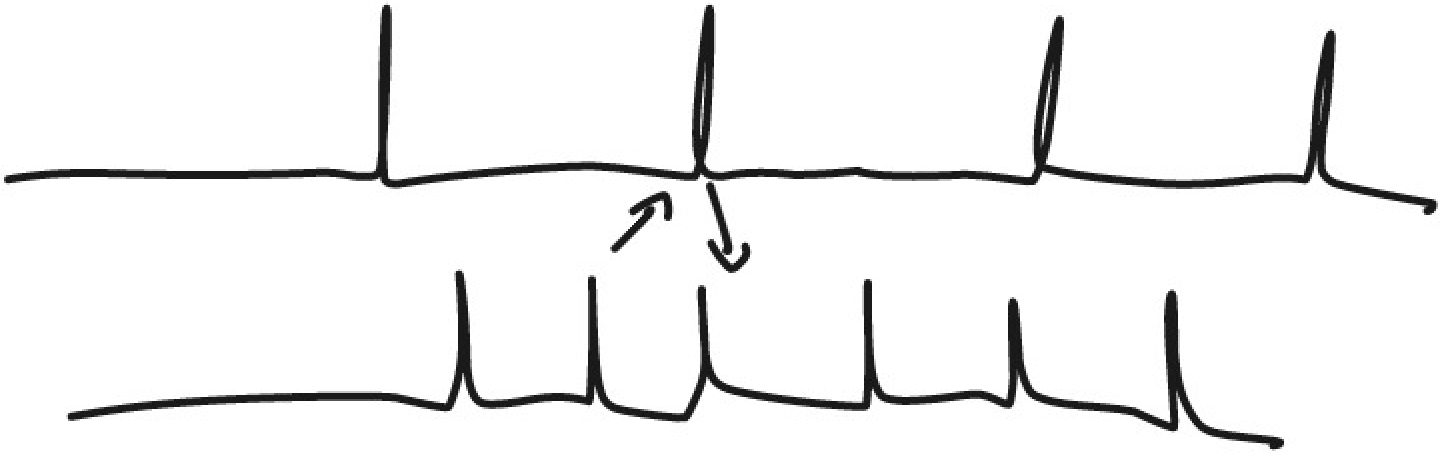
$\delta\omega > K$ no fixpoints

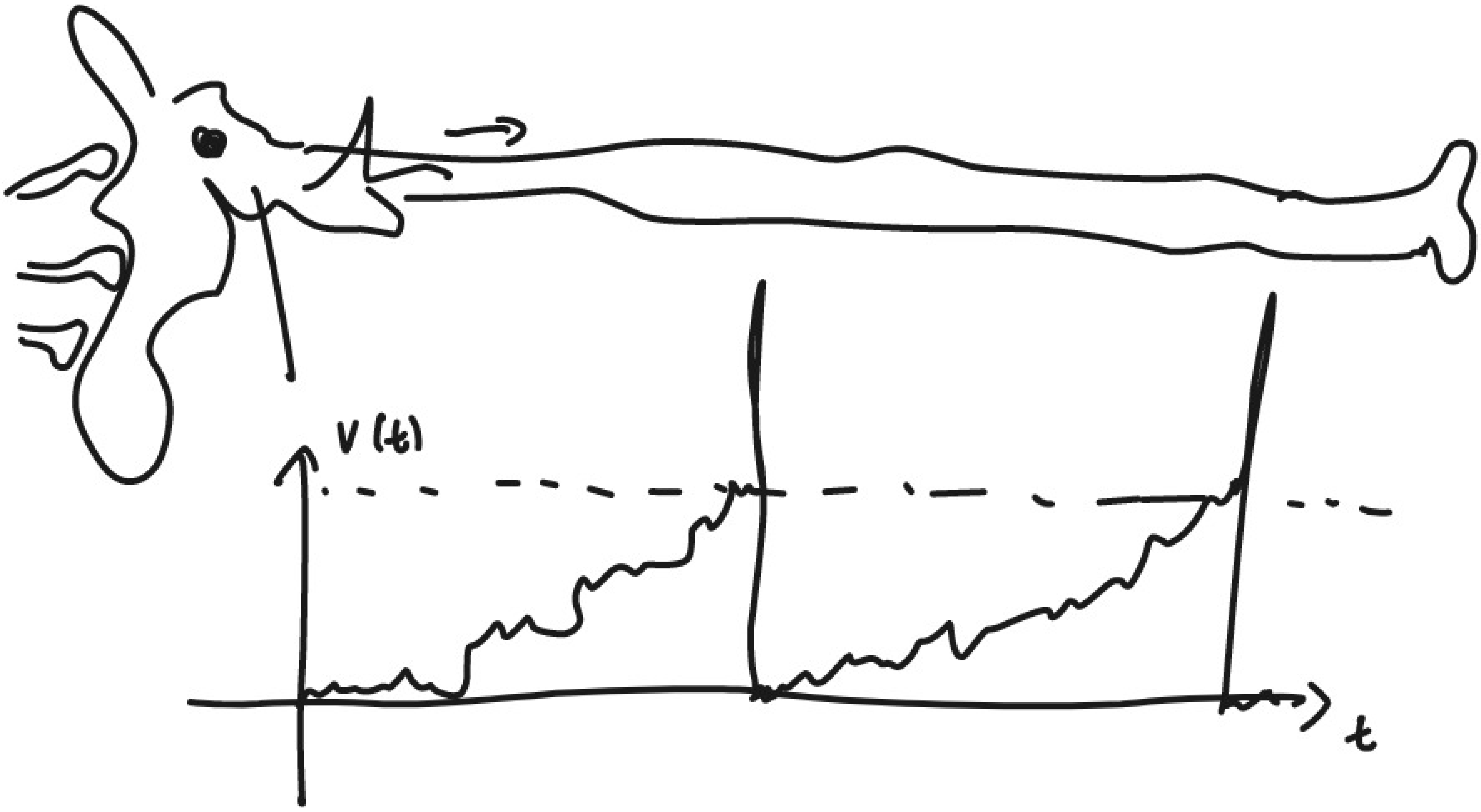
$\delta\omega < K$ one stable fixp.

$\delta\omega = K$

$$\boxed{\begin{array}{ll} \frac{\delta\omega}{K} > 1 & \text{no sync.} \\ \frac{\delta\omega}{K} < 1 & \text{sync.} \end{array}}$$

pulse coupled oscillators





$x(t)$

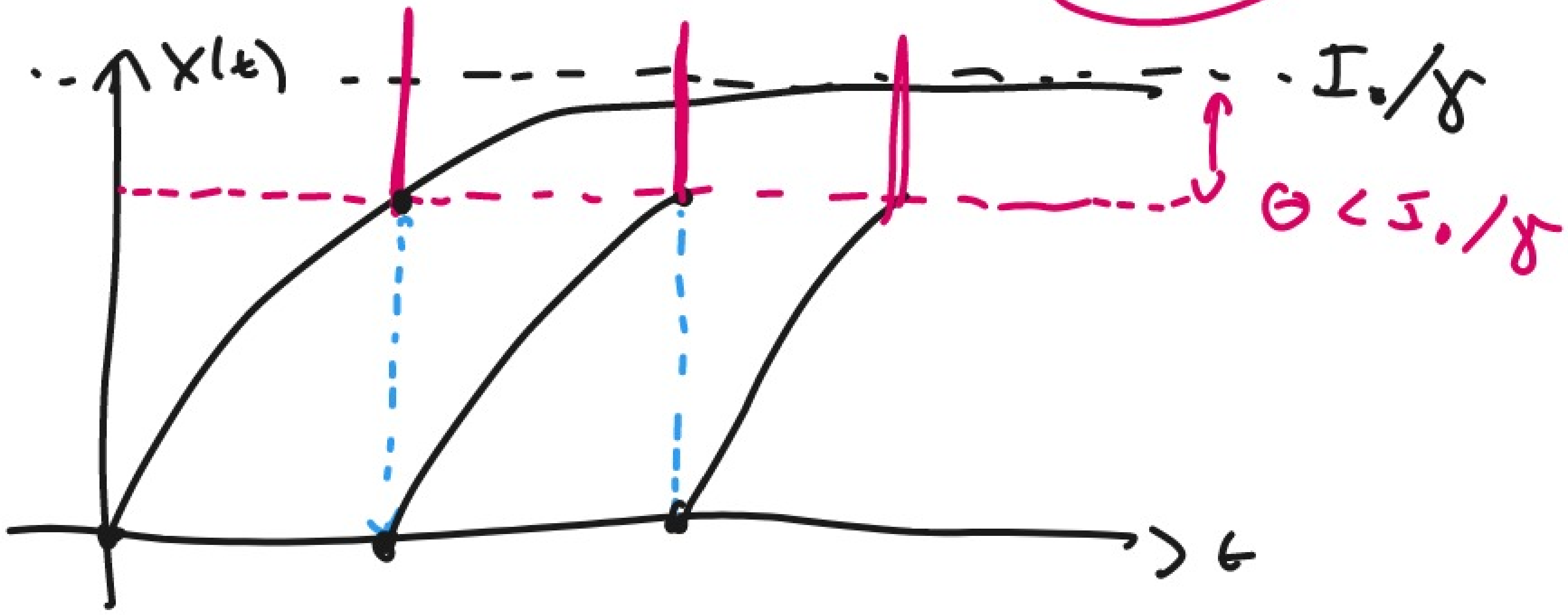
$$\boxed{\dot{x} = I_0 - \gamma x}$$

$x^* = I_0/\gamma$ stable

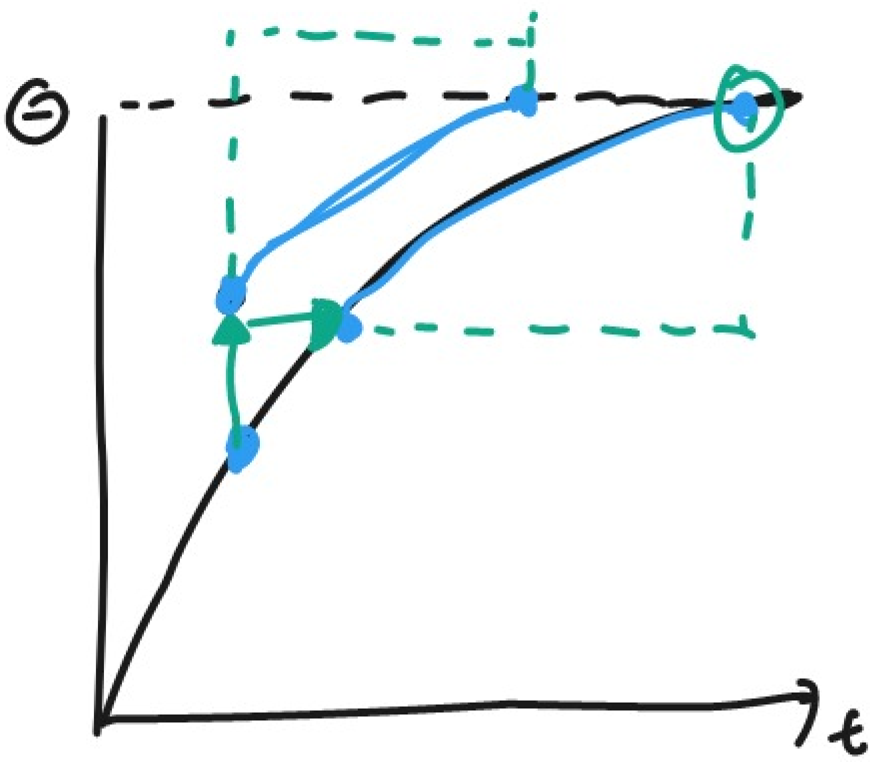
$x(t)$

$\dot{x} = I_0 - \gamma x$

$x^* = I_0/\gamma$



1



2

Δt