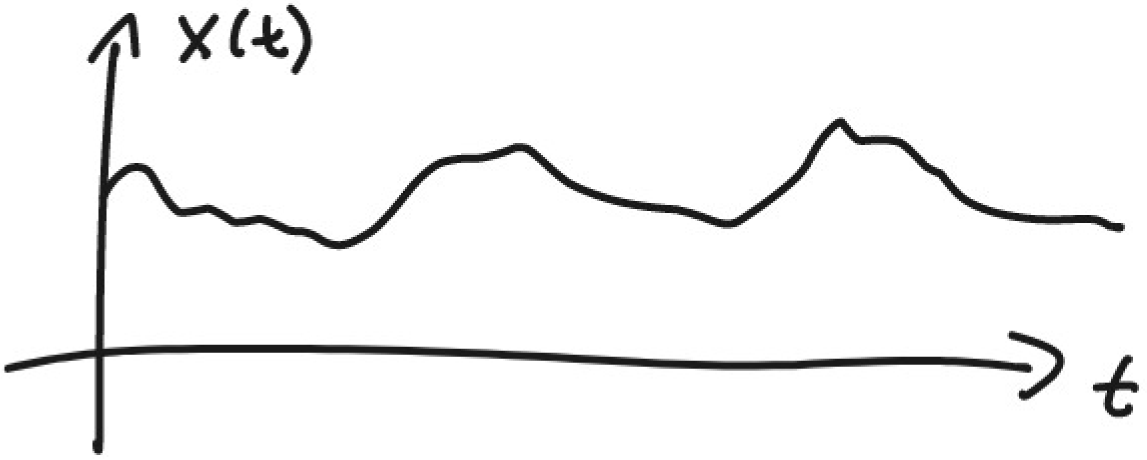
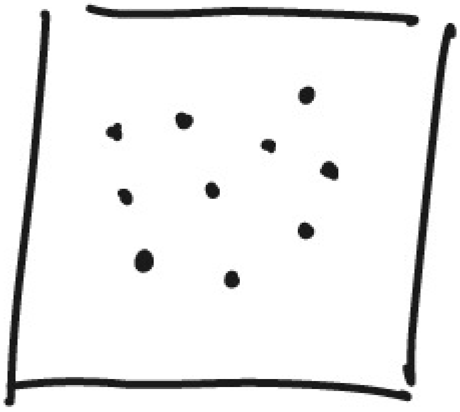
Goodmorning

$$\boxed{x_{n+1} = \lambda x_n (1 - x_n) = f(x_n)}$$

$n = 0, 1, \ldots$



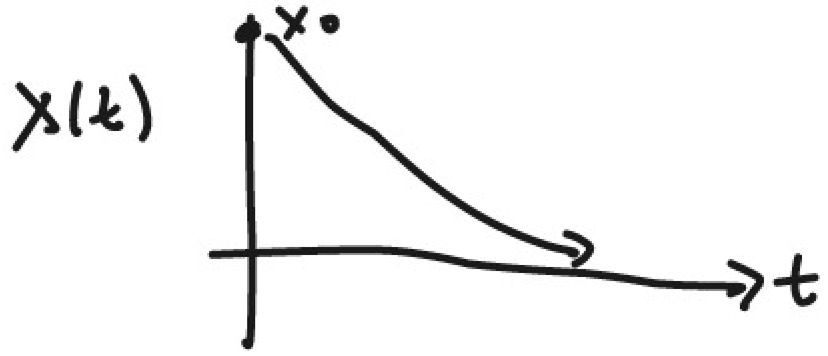
$$\frac{dx}{dt} = f(x) \qquad \boxed{\dot{x} = f(x)}$$



A $\qquad \boxed{A + A \to \emptyset}$

$N \gg 1$

$x(t) = \#$ particles



$$x(t) = x_0 e^{-\alpha t}$$

$$x(t) = 1/t^{17}$$

$$x(t) = x_0 e^{-\sqrt{t}}$$

$$\boxed{x(t+\Delta t) = x(t) + \Delta t\, f(x(t))}$$

$$\frac{x(t+\Delta t) - x(t)}{\Delta t} = f(x(t))$$

$$\frac{dx}{dt} = f(x)$$

$$A + A \to \emptyset$$

$x(t)$ $\qquad$ $\Delta x = ?$

$$\boxed{\Delta x \sim -x \cdot x}$$

x particles contribute

prob. of encounter

$$\Delta x = -\Delta t\, \alpha\, x^2$$

$$\boxed{\dot{x} = f(x) = -\alpha x^2 = -\alpha x^2}$$

$$\boxed{\frac{dx}{dt} = -\alpha x^2} \rightsquigarrow \frac{-dx}{\alpha x^2} = dt$$

$$-\int \frac{dx}{\alpha x^2} = t - t_0$$

$$\frac{1}{\alpha x}\Bigg|_{x_0}^{x(t)} = t - t_0$$

$$\boxed{x(t) = \frac{1}{1/x_0 + \alpha(t - t_0)} \sim \frac{1}{t}}$$

$$\mathrm{I}: \underline{A + A \xrightarrow{\alpha} \phi} \qquad \underline{\mathrm{II}}: \underline{A \xrightarrow{\beta} 2A}$$

$$\dot{x} = \frac{dx}{dt} = f(x)$$

$$\Delta x_{\underline{\mathrm{I}}} = -\alpha x^2 \Delta t \qquad \Delta x_{\underline{\mathrm{II}}} = \Delta t \beta x$$

$$f(x) = -\alpha x^2 + \beta x$$

$$f(x) = x(\beta - \alpha x)$$

$$\boxed{\dot{x} = x(\beta - \alpha x)}$$

reproduction (β)

annihilation (αx)

$\Rightarrow$ $x(t)$

$$\dot{x} = f(x)$$

$$\dot{x} = \boxed{0 = f(x^*)}$$

$x(t) = x^*$

$f(x^*)$ x^* = fixpoints

$$\dot{x} = x(\underbrace{\beta - \alpha x}) = f(x)$$

$f(0) = 0$ $x^* = 0$

$\rightarrow$ $x^* = \beta/\alpha$ $\Rightarrow$ $\dot{x} = 0$

$$\dot{x} = x(\beta - \alpha x) = f(x)$$

$x^* = 0$

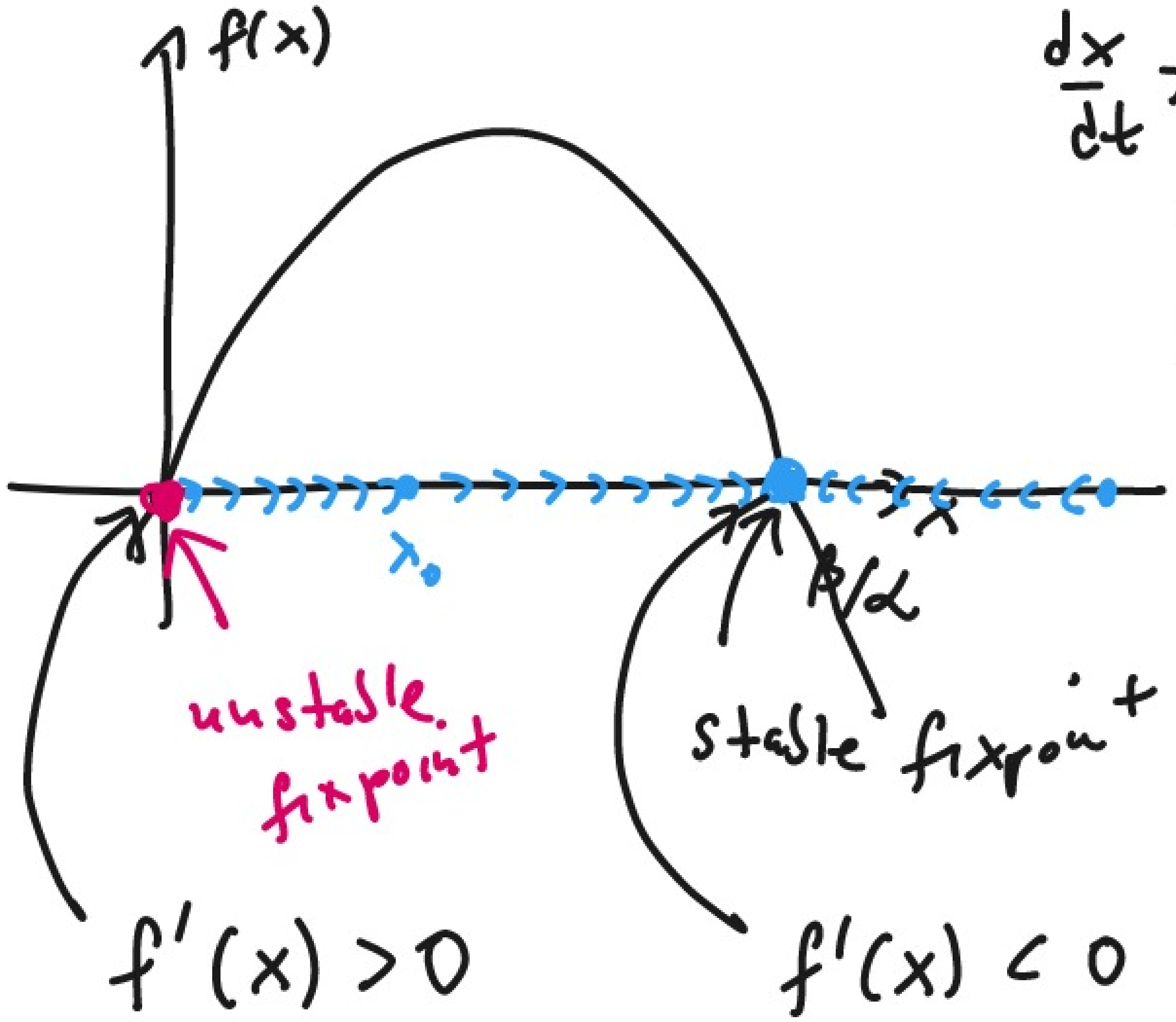
$x^* = \beta / \alpha$

$\frac{dx}{dt} > 0$

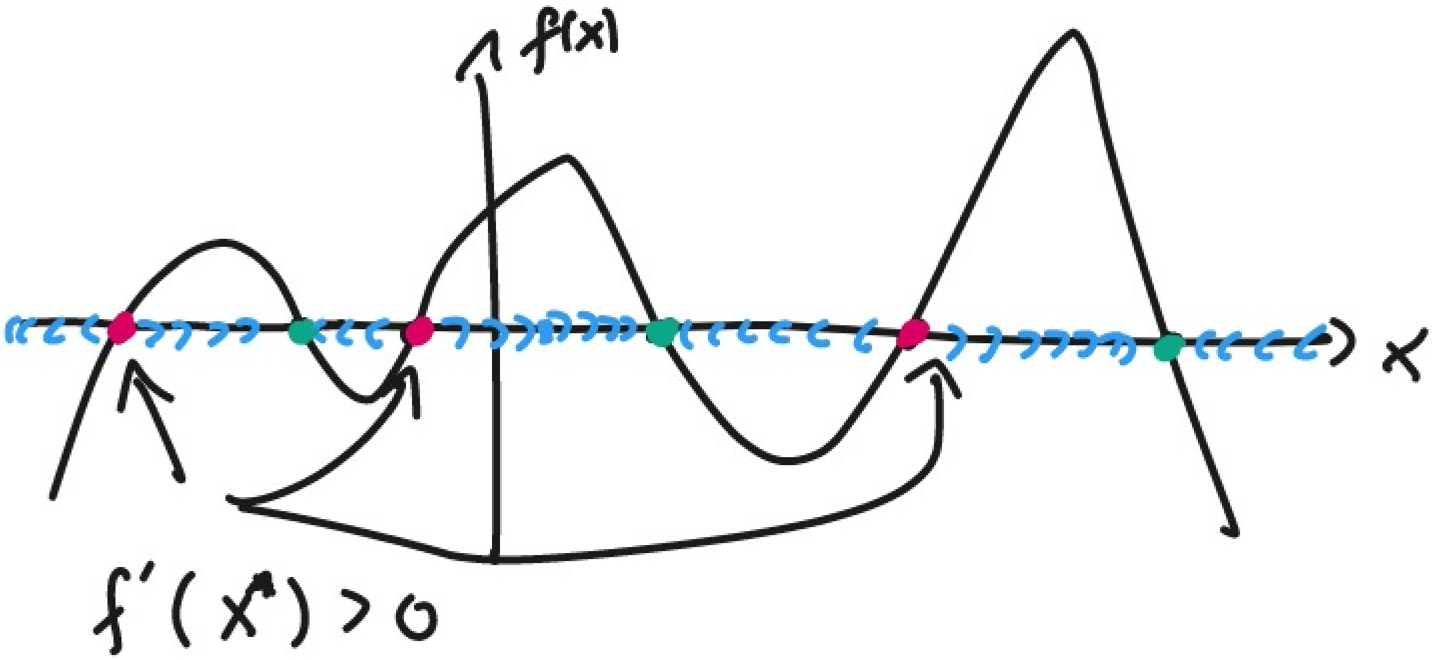
$f(x) > 0$

$\frac{dx}{dt} < 0$

$f(x) < 0$



$$\dot{x} = f(x)$$



$$\dot{x} = x(\beta - \alpha x) = f(x)$$

$$f(x) = \beta x - \alpha x^2$$

$x^* = 0$

$x^* = \frac{\beta}{\alpha}$

$$f'(x) = \beta - 2\alpha x$$

$$f'(0) = \beta > 0$$

$$f'(\beta/\alpha) = \beta - 2\alpha \cdot \frac{\beta}{\alpha} = -\beta < 0$$

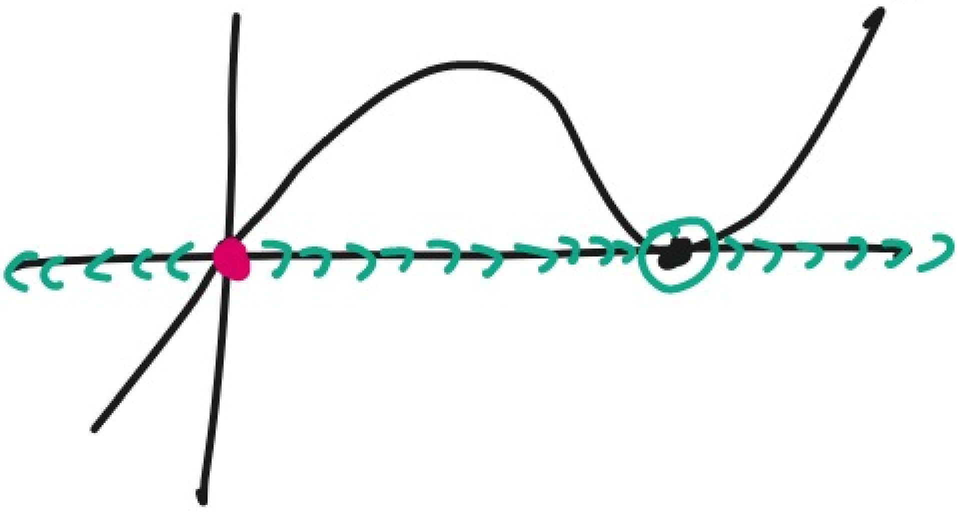
$$\dot{x} = x(x-1)^2 = f(x) = x[x^2 - 2x + 1]$$

$$= x^3 - 2x^2 + x$$

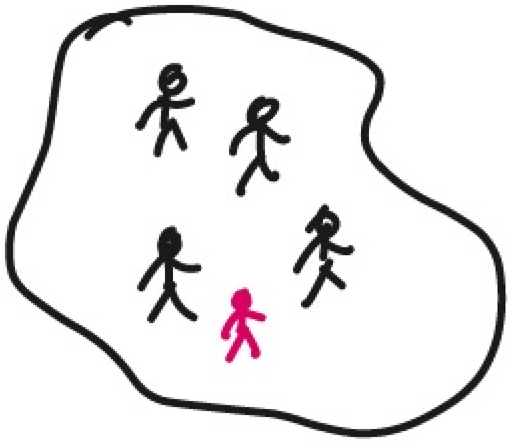
$f(x) = 0$?

$x^* = 0$

$x^* = 1$



SIS - Model



susceptibles

$$N = S(t) + I(t)$$

infecteds

$$I + S \xrightarrow{\alpha} 2I \qquad (1)$$

$$(2) \qquad I \xrightarrow{\beta} S$$

$I(t)$

$$\Delta I_{(2)} = -\Delta t \, \beta \, I \quad ✓$$

$$\Delta I_{(1)} = \Delta t \, S \cdot \frac{I}{N} \, \alpha \quad ✓$$

$$\boxed{\dot{I} = \alpha S \cdot \frac{I}{N} - \beta I} \qquad S = N - I$$

$$\boxed{\dot{I} = \alpha (N - I) \cdot \frac{I}{N} - \beta I}$$

$$x = \frac{I}{N} \qquad \dot{x} = \frac{\dot{I}}{N} = \alpha (1 - x) \cdot x - \beta x$$

$$\dot{x} = \alpha x(1-x) - \beta x = f(x)$$

$$\dot{x} = x\underbrace{\left(\alpha(1-x) - \beta\right)} = f(x)$$

$\boxed{x^* = 0}$

$\hookrightarrow$ $\alpha(1-x) = \beta$ $\rightsquigarrow$ $\boxed{x^* = 1 - \frac{\beta}{\alpha}}$

$\hookrightarrow$ only exists if $\boxed{\alpha > \beta}$

$R_0 = \alpha/\beta$ basic reproduction ration

$R_0 > 1$ $x^* = 1 - \frac{1}{R_0}$ exists

$R_0 < 1$ it doesn't.

$A + A \to \phi$ $\qquad$ $A \to 2A$

$$\dot{x} = x(\underbrace{\alpha - \beta x}_{})$$

$S + I \to 2I$ $\qquad$ $I \to S$

$$\dot{x} = x(\alpha(1-x) - \beta)$$