Klausur zum Modul

# Komplexe Systeme in der Biologie

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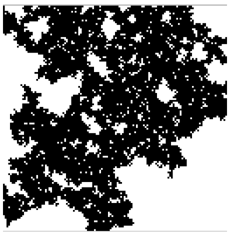
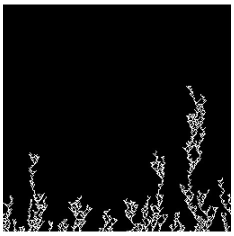
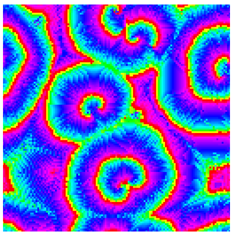
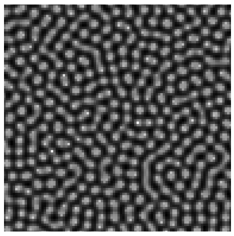
- On every problem you can up to 12 or 16 points, the maximum score for the test is 100 points.
- Write your name and matrikelnummer on every sheet of paper (lower left corner).
- If you need additional sheets of paper ask for them.
- Calculators are neither required nor allowed.

1. [12 pts.] Given the adjacency matrix $\mathbf{A}$ shown below for a symmetric unweighted network
   a) draw the network
   b) rank the nodes according to degree
   c) rank the nodes according to clustering coefficient (the clustering coefficient for nodes with degree 1 is 0).
   d) The matrix $\mathbf{B} = \mathbf{A}^3$ is shown below as well. What do the diagonal elements of $\mathbf{B}$ measure? Explain and confirm in your sketch of the network.

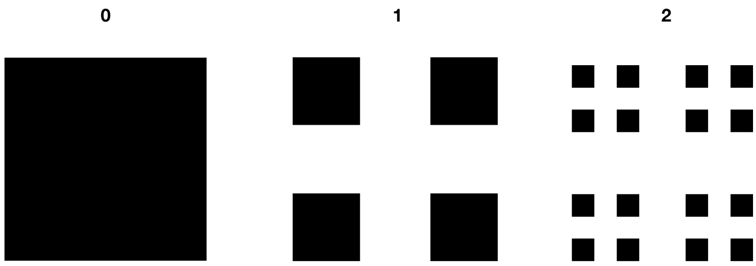
$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 2 & 3 & 6 & 2 & 2 & 1 & 1 & 2 \\ 3 & 2 & 6 & 2 & 2 & 1 & 1 & 2 \\ 6 & 6 & 4 & 6 & 8 & 8 & 0 & 1 \\ 2 & 2 & 6 & 2 & 4 & 2 & 1 & 2 \\ 2 & 2 & 8 & 4 & 2 & 1 & 2 & 5 \\ 1 & 1 & 8 & 2 & 1 & 0 & 3 & 5 \\ 1 & 1 & 0 & 1 & 2 & 3 & 0 & 0 \\ 2 & 2 & 1 & 2 & 5 & 5 & 0 & 0 \end{pmatrix}$$

2. [12 pts.] Below you see four temporal snapshots of two-dimensional dynamical systems that were generated by four different processes listed in random order below and denoted by a, b, c and d. What mechanism generated what pattern?

   a) Turing mechanism
   b) a stochastic cellular automaton
   c) A percolation system near the critical point
   d) A spatial system of three cyclically inhibitory interacting species.

3. [12 pts.] Consider the following geometric structure that is generated iteratively. You start with a filled square and remove a cross from the middle such that 4 smaller pieces remain. The area of the remaining pieces is 1/9 of the original piece each. You continue to do this ad infinitum. Compute the fractal dimension of the object that you get.



4. [12 pts.] Analyse the following dynamical systems **graphically**, find their fixpoints and determine their stability.

   a) $\dot{x} = x^2 - 2x^3 + x^4$

   b) $\dot{x} = g(x)h(x)$ where $g(x)$ has two fixed points $x_1$ and $x_2$ and $h(x)$ has two fixed points $x_3$ and $x_4$. Plus we assume that $x_1 < x_3 < x_2 < x_4$ (Achtung!! look at the indices, this is not a typo!!). You also know that both $g(x)$ and $h(x)$ are positive on the interval between their fixed points and their derivative is non-zero at the fixpoints.

5. [16 pts.] Consider the dynamical system for positive state variable $x$ and $y$.

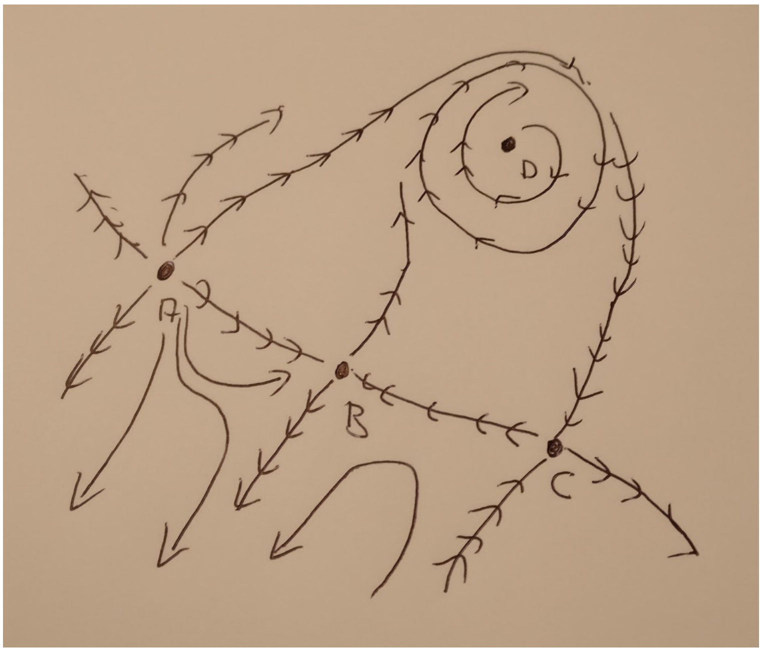
$$x = x(1-y)$$
$$y = y - 1/x$$

a) Compute the fixed point of the system
b) Compute its stability
c) Classify the fixed point
d) Draw the null-clines of the system
e) Sketch some possible trajectories of the system

6. [12 pts.] A system $x = f(x; \lambda)$ has a stable fixed point for $\lambda < 1$. Crossing the critical value $\lambda = \lambda_{c,1} = 1$ the system undergoes a pitchfork bifurcation. When $\lambda$ reaches a second critical point $\lambda_{c,2} = 3$ the system has a saddle node bifurcation. For very large $\lambda$ the system only has one stable stationary point (some other bifurcartions may have occured in between). Draw two possible bifurcartion diagrams that fullfill these requirements.

7. [12 pts.] Consider the phase-diagram in the sketch below. The dynamical system has 4 fixed-points (A,B,C and D). Someone computed the eigenvalue pairs for each fixed point for you. Now you have to figure out which eigenvalue pair belongs to each fixedpoint. Do it.

- 1: $\lambda_{1/2} = +1/3 \pm i/4$
- 2: $\lambda_1 = 1$, $\lambda_2 = -3$
- 3: $\lambda_1 = -e^{-i\pi}$, $\lambda_2 = e^{i\pi}$
- 4: $\lambda_1 = \sqrt{2}$, $\lambda_2 = -i\sqrt{-2}$



8. [12 pts.] Below is the core of a netlogo program.

   a) What does it simulate?

   b) What different outcomes do you expect when parameters $\alpha$ and $\beta$ are varied?

   c) Illustrate in a drawing what will happen here.

   d) How would you set up a simple dynamical system (neglecting the spatial extent) that can describe this dynamical system roughly?

```
patches-own [state]

to setup
  ca
  ask patches [set state 0]
  ask patch 0 0 [set state 1]
  reset-ticks
end


to go
  ask patches with [state = 1] [
    ask one-of neighbors [
      if state = 0 and random-float 1 < alpha [ set state 1 ]
     ]
  ]
  ask patches with [state = 1] [
      if random-float 1 < beta [ set state 0 ]
  ]
  tick
end
```