Klausur zum Modul

Komplexe Systeme in der Biologie

Prof. Dirk Brockmann

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Institut für theoretische Biologie - Institut für Biophysik

Institut für Biologie

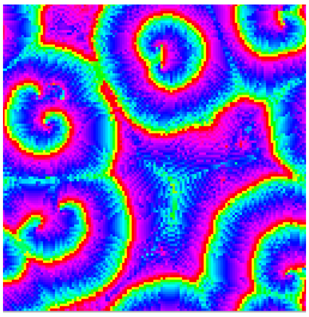
Fakultät für Lebenswissenschaften

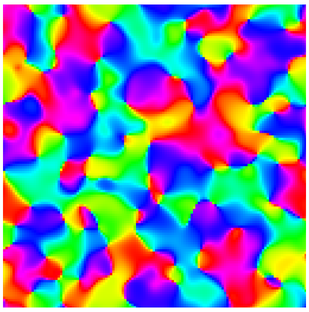
Humboldt Universität zu Berlin

17. Oktober 2016 - 18:00 - 20:00 Uhr - Bearbeitungszeit 90 min.

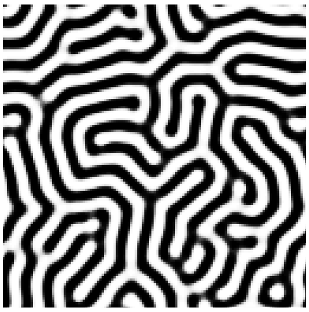
- On every problem you can earn 10 points, the maximum score for the test is 100 points.
- Write your name on every sheet of paper (lower left corner).
- If you need additional sheets of paper ask for them.
- Calculators are neither required nor allowed.

1. [10 pts.] Below are two images of spatially coupled oscillators, one is a system of phase coupled oscillators, the other of pulse coupled oscilators. Identify which pattern belongs to which system and explain your answer.

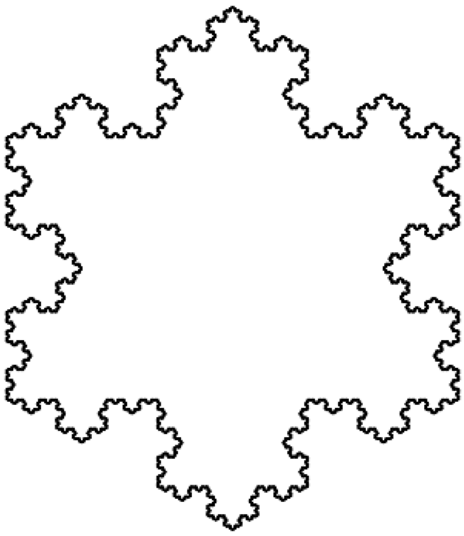




2. [10 pts.] Below is a striped stable pattern. Explain **two** mechanisms by which this pattern could have emerged.



3. [10 pts.] Below is an image of the Koch-Snowflake. Explain an iterative process that generates this structure and compute its fractal dimension.



4. [10 pts.] Analyse the following dynamical systems graphically, find their fixpoints and determine their stability.

   a) $\dot{x} = x^2 - 2x$

   b) $\dot{x} = -(x-a)(x-b)(x-c)$ with $a > b > c > 0$ as constant parameters

5. [10 pts.] Consider a system of two coupled oscillators with phase variables $\theta_1(t)$ and $\theta_2(t)$ and natural frequencies ω_1 and ω_2. The dynamical system that governs their dynamics and interaction is defined as

$$\begin{aligned} \dot{\theta}_1 &= \omega_1 - \frac{K}{2}\sin(\theta_2 - \theta_1) \\ \dot{\theta}_2 &= \omega_2 - \frac{K}{2}\sin(\theta_1 - \theta_2) \end{aligned}$$

where $K \geq 0$ is the coupling strength between the oscillators.

a) What is the asymptotic state of the system when $\omega_1 = \omega_2$

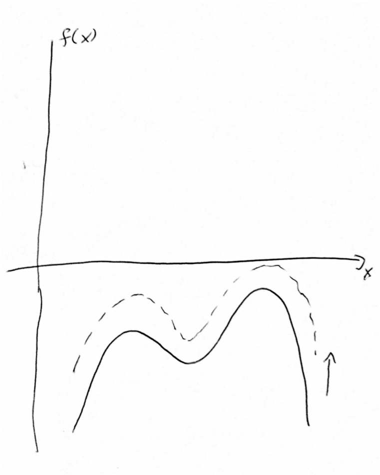
b) If $\omega_2 \neq \omega_1$ what is the critical coupling strength K such that the oscillators synchronize and what is the phase difference in the synchronous state.

6. [10 pts.] Construct a two-dimensional activator-inhibitor system that has one stationary state that is asymptotically stable.

7. [10 pts.] Below is a sketch of a function $f(x)$. Now consider the dynamical system

$$\dot{x} = g(x) = f(x) + \lambda$$

Sketch a bifurcation diagram of the dynamical system and classify the bifurcations



8. [10 pts.] Consider a two dimensional dynamical system $\dot{x} = f(x, y)$, $\dot{y} = g(x, y)$. Assume that it has four fixpoints $P_1 = (0, 0)$, $P_2 = (1, 1)$ and $P_3 = (-1, 1)$ and $P_4 = (0, -1)$. You also know that the system has a stable limit cycle somewhere. When investigating the fixpoints' stability you find the following pair of eigenvalues

   - for P_1: $\lambda_{1/2} = +1 \pm i$
   - for $P_2 : \lambda_1 = 1$, $\lambda_2 = -1$
   - for $P_3 : \lambda_1 = 1/2$, $\lambda_2 = -2$
   - for $P_4 : \lambda_1 = 1/2$, $\lambda_2 = -1$

   a) Classify the fixed points

   b) Sketch a hypothetical phase portrait of a dynamical system that has these properties

   c) Sketch a few hypothetical trajectories

9. [10 pts.] The Viszek model is the simplest model for collective behavior of moving agents. Explain how the model works. What are the parameters, what are the dynamical variables and what are the behaviors observed? How would you define an order parameter that can quantify the collective state of the system.

10. [10 pts.] Assume you have a system of interacting species A and B that live in a two-dimensional world in which they move around diffusively. Inititally all individuals are scattered randomly in this world. The interact by the following rules

- When A encounters B they fight and one of them wins
- When A encounters another A they reproduce, the same holds for B
- There's a global factor that ensures that the world will not be overcrowded, e.g. species die when the density is too high

a) Describe qualitatively the spatial pattern that is expected to emerge in such a system.

b) Sketch it.